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A Review on Quantitative Models in Railway Rescheduling

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Abstract— This paper draws an overview of various quantitative rescheduling models that have been developed in past research in dealing with delay management of passenger railway services. In general, the quantitative models highlighted aimed to reduce or minimize the service delays experienced by passengers when service disruptions occur. In many literatures observed, the delay management problems are formulated as mathematical and non-mathematical models, which are then tested on real-life data of railway companies worldwide. Local search and exact methods which provide near-optimal and optimal solution**s**, respectively, are adopted as the solution approaches, with a target of producing short computing time and better quality solution. In this sense, this review paper provides a comprehensive analysis for researchers to enhance the existing rescheduling models or perhaps to adopt a new approach in solving the models developed. This review also suggests the need to minimize service delays for passenger trains due to disruptions, in the context of local railway services.

Index Terms— Delay management, quantitative models, rail passenger, rail service delays, railway rescheduling

1 INTRODUCTION

IN rail transportation systems, operational problems and unexpected events such as technical failures, equipment breakdown, extraordinary passenger volumes, track accidents and/or weather conditions can occur and causing disruption to certain segments of the railway network. Disruption may cause the control managers to reshuffle train orders, make unplanned stops and break connections, re-route trains and even delay or cancel scheduled services. Changes in the original train departure and arrival schedules can create conflicts in the use of tracks and platforms, thus, even what originally presumed as a minor disruption at certain point of a railway network is capable of instigating further propagation of disturbances throughout the railway system. Thus, railway rescheduling decisions must involve finding immediate solution to the new assignment problem of allocating, sequencing and synchronizing the utilization of resources, i.e., tracks, crew and train, to minimize the effect of railway traffic perturbations.

This paper intends to review the literature studies on the quantitative rescheduling techniques used in delay management of passenger railway services. The main contribution of the review is to provide a comprehensive analysis for finding a better rescheduling model to solve the problem of passenger rail service disruptions. The paper is also expected to highlight the future research direction as the findings of the research could bring new ideas for multiple perspective improvement, not only in the field of delay management, but also in business engineering process and quality engineering improvement.

This review paper is outlined as follows: Section 2 presents some quantitative models that have been used in delay management, specifically mathematical and non-mathematical programming models. Section 3 briefly discusses the mathematical solution approaches used to solve the models, together with some potential approaches that may be used in solving future mathematical models. Section 4 presents the final remark and future direction of the study.

2 QUANTITATIVE MODELS OF SERVICE DELAY MANAGEMENT

Due to the numerous models proposed in the literatures in formulating the problem of delay management, this paper will only focus on the quantitative models, which can be classified as mathematical and non-mathematical models.

2.1 Mathematical Programming Models

A delay management problem was first formulated as a mixed integer programming (MIP) based on event-activity networks by Schobel [1]. It deals with the delay involved in transportation network. The objective of the model developed is to minimize the sum over all delays of all customers in the network, and it has chosen linear programming to formulate the problem. The study, however, looked at general types of transportation delays, not only train delays.

Later, the study was extended to a railway service whereby this time a priority decision was added to the integer programming (IP) model. As the rail track system is subjected to a limited capacity, the priority decisions would decide the sequence of trains in passing a track [2]. A reduction technique derived for the network is used as the basis to solve the capacitated delay management problem using exact and

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heuristics approaches. The work contributes a significant finding as the four heuristics suggested are able to yield a faster result than the optimal solution obtained from IP formulation.

Still focusing on the capacity constraints in the mathematical formulation, a new integrated model was later developed based on macroscopic model which contain the microscopic capacity constraints. Its main objective is to minimize train delays and dropped connections. It was solved using the branch and bound (B&B) approach by ordering the related activities in branches [3]. In some special cases, the upper bound and the lower bound can be found using heuristics and LP-relaxation methods, respectively. By examining the result in four cases, the solution approach yields a good result in terms of the sum of delays.

Capacitated IP model was also put forward by Flier et al. [4] who relaxed the assumption of fixed rolling stock circulations and allowed vehicle schedule changes in the new model. Event-activity network which is constructed from set-partitioned problem was proposed. To solve the problem, heuristics and B&B approaches were suggested. The efficiency of the solution, however, is still under research.

Tornquist and Persson [5] presented a model and solution approach for the railway traffic rescheduling problem with a highly complex setting, taking into account the large number of railway tracks and segments and each rail network direction. An MIP model representing the disturbed *n*-tracked network is solved using CPLEX software by means of B&B solution procedure. Four strategies were evaluated by their optimality, speed and sensitivity to the problem size. The third strategy which allows specific number of order swaps for specific segments gave the best results in many cases, in terms of the fast processing time and better solution quality.

Acuna-Agost et al. [6] proposed two models namely an MIP model and a Constraint Programming (CP) model, which shares common objective functions that are to minimize delays, changes of tracks and platform and unplanned stops. Both models differ in terms of decision variables and constraints significantly. The proposed MIP is found to have developed very efficient solution methods but it utilizes more memory as compared to CP, as large number of binary variables is needed to model the order of trains. The authors describe the complexity of these two models due to allocation of tracks and platforms, connection between trains, bidirectional/multi-track lines and extra time for accelerating and braking were also considered.

The study was then extended to seek for a more practical method in a larger scale problem, whereby a new statistical method called Statistical Analysis of Propagation of Incidents (SAPI) was introduced [7]. In order to reschedule trains under disrupted operations, the technique analyzes the factors that could contribute to the consequences of service disruptions, followed by reducing the search space of the inherent optimization problem. Upon investigation, SAPI was found to outperform other MIP-based methods used in the study.

Tornquist & Persson [8] proposed a two-level formulation to solve rail traffic conflicts which are due to disruptions. The lower level is a linear programming (LP) model, while the upper level is the tabu search and simulated annealing that determine the order of trains on blocks. With an objective to minimize the total delay of the entire traffic system, the tabu search approach yield a better result when compared to the optimum solution. The study, however, could be extended to heterogeneous data set with more interacting traffic, so as to analyze the practicality of the model and solution approach.

When schedule is disrupted, the rescheduling process, involving train and crew scheduling, can be done manually by experienced dispatchers. However, for a large-scale railway network, this manual task can be very difficult and time-consuming. Rezanova and Ryan [9] worked on a train driver recovery problem (TDRP) to find the set of feasible train driver recovery duties for drivers in disruption situation, while keeping the modifications on the original schedule at minimum level. The TDRP is formulated as a set partitioning problem and a solution approach is based on solving the LP relaxation with constraint branching strategy using the B&B tree search. Any fractional solutions will indicate that more than one driver is competing for one or more train tasks in their recovery duties. When tested on the Danish passenger railway operator DSB S-tog A/S data, which has a very limited number of train tasks, the optimization method yields integer solutions within a few seconds and subsequently solve the train deriver recovery problem. However, the basic solutions to the TDRP-LP may not turn out to be natural integers if applied to other railway operators which have many train tasks that a driver is competing for.

A study on the congested Japanese railway lines was done by Sato et al. [10], who developed a 0-1 integer programming formulation based on a network flow model for crew rescheduling problem (CRP) and the vehicle rescheduling problem (VRP). The study intends to minimize the differences between the new and original schedules when disruption occurs to train schedule. The solution concept is based on two-phase modification of temporary solutions, which uses heuristic flow modification and a local search technique. Besides being able to design interactive operations between users and the system, the proposed model can also provide a highly responsive solution for the real-world vehicle rescheduling problem within a limited time period. With a list of superiority comparison between the original crew and vehicle schedules and the proposed CRP/VRP schedule, the latter is more practical and easily implemented in developing a support system for train recovery operations.

Another train crew rescheduling study in Japan is also highlighted by Sato and Fukumura [11], who formulated driver rescheduling problem as an IP model with set-covering constraints. Column generation technique is used to solve the optimization algorithms by relaxing the integer constraints. Computational experiments on real data yield a good solution within a short computing time. The findings of the study, however, may not be applicable to many train service disruption problems since it assumes that the information regarding the service delays and cancelations are always known in advance.

A novel event-based IP model which incorporates shifting and canceling of trips and modifying the circulation of train schedules is proposed by Fekete et al. [12]. The study aims to maximize the number of recovered trips in the new timetable in the event of service disruptions. Some reduction techniques are used to reduce the size of the IP so as to accelerate the computing time. Four scenarios of service disruptions were tested on Vienna subway system, in which feasible and fast solutions are obtained for most of the cases. The method proposed is in fact a very practical approach in a large-scale problem due to its good solutions, in terms of fast response, high utilization of tracks and its tested optimal and near-optimal solutions.

A study on railway delay management problem is done at different perspective, by Kliewer and Suhl [13]. They choose to evaluate and compare two dispatching strategies when service disruptions occur. The first strategy is based on optimization technique, which adopts a mathematical programming algorithm, while the second strategy is based on heuristic rules. Targeting to minimize the unexpected passenger waiting time during disruptions, a series of test was done on Deutsche Bahn of German railways. The results show that the heuristic strategies provide better outcomes in terms of the passenger waiting time and also when the disturbance density is high. This comparative study involving the same set of data with two different methods provides a good literature in the study of delay management, which rarely considered in many academic researches.

TABLE I

OVERVIEW ON RELATED WORKS USING INTEGER AND MIXED INTEGER PROGRAMMING MODELS

Publications	Research objectives	Results	Models tested on real data from:
Fekete et al. (2011)	To maximize the no. of recovered trips, possibly with delays.	A dispatching timetable for rescheduling of trips and vehicle circulations	Vienna subway line
Sato and Fukumura (2010)	To speed up the process time for rescheduling and reduce dispatchers' workload.	An adjusted timetable and driver rescheduling plan in disrupted operations	Japan Freight Railway company
Acuna Agost (2010)	To minimize cost of delays and cost of final delays.	A new provisional timetable that minimizes the total impact of disruptions	Railway network in France and Chile
Acuna Agost et al. (2009)	To minimize the rescheduling cost and impact for passengers under disrupted operations.	A new provisional timetable that supports complex rules and constraints	French railway network
Anita Schobel (2009)	To minimize average delay with capacity constraint.	A disposition timetable for wait-depart decisions with limited track capacity	Germany Deutsche Bahn
Sato et al. (2009)	To minimize differences between the new and original schedules.	Two-phase solution approach for crew and vehicle rescheduling problem	Japanese Railway line
Flier et. al (2008)	To minimize the sum of all delays, allowing rolling stock circulations.	A disposition timetable that re-optimizes the original vehicle schedules	Germany Deutsche Bahn
Schachtebeck and Schobel (2008)	To minimize average delay with priority decisions.	A disposition timetable with priority decisions for capacitated case	Germany Deutsche Bahn
Tornquist & Persson (2007)	To minimize service delays and costs in rescheduling railway traffic.	A rescheduling solution for large, fine-grained and high interacting railway traffic network	Sweden railway network
Tornquist & Persson (2005)	To minimize the total delays in railway traffic due to disturbances.	A traffic rescheduling solution for handling trains meets and overtakes on railway track sections	Sweden railway network
Anita Schobel (2001)	To minimize the sum of all delays over all customers.	A schedule to make wait-depart decisions for multiple vehicles in transportation network	Germany Deutsche Bahn

Table I presents the summary of the relevant works that use IP and MIP models in railway traffic rescheduling. The research deliverables in the form of new schedules and timetables are presented explicitly next to the research objectives column, whereby majority is concerned with the aim to minimize the average delays and total delays. The right most column shows the railway companies worldwide which provide the real-life data for the models' validity experimentation, with Europe rail operators dominating the studies.

2.2 Non-mathematical Programming Models

As train rescheduling is a large size combinatorial problem and requires high immediacy, it is difficult to model a single rescheduling problem which is applicable to many disruption instances. Therefore, targeting on a fast response solution, Norio et al. [14] combines a simulated annealing with Program Evaluation and Review Technique (PERT) to come up with a train rescheduling algorithm, taking passengers' dissatisfaction as the objective criteria. The meta-heuristics approach yield a satisfactory result of one minute execution time when tested on two types of service disruption using real data of Tokyo railway line. However, the model should be further tested on various other types of service disruptions so that the applicability of the algorithm can be critically explored.

A macroscopic timetable decision support tool is presented by Corman et al. [15] which is intended to solve conflicts between consecutive trains for large-scale network and modeling feasible headway distances in complex station interlocking areas. The tool which is called Conflict Detection and Resolution (CDR) system, has a combinatorial structure of a job shop scheduling problem. The objective of the problem formulation is to minimize the maximum delay at each point of the studied area. Feasible solutions which are computed using B&B algorithm are expected to be obtained if all constraints are satisfied and the network is deadlock-free. Experiments done on Dutch railway network shows that the aggregated formulation which aggregates track sections into station routes when dealing with sectional-release route locking has yield a good result in terms of delay minimization and processing time. This support system offers an advanced technological contribution to railway traffic controllers in handling rail traffic disruptions.

Garcia et al. [16] proposed an online optimization model to deal with real time decision and control in public transport systems under disruptions, with the objective to minimize the waiting time on the whole network. Based on a series of numerical tests which were done on the regional train network of Madrid, it is found that this new heuristic algorithm, which is developed based on a predictive simulation and a queue model, has successfully reduced the total expected waiting time in the transport system. According to the tests, even though for many cases short execution times were observed, the execution time is a little longer when the disturbed line is too far from the assisting lines. Some modifications may be needed to entertain this shortcoming, possibly by decreasing the tolerance bandwidth used to compensate response time.

From a general public transport system, the work was then extended to a specific case of the Vehicle Re-Scheduling Problem (VRSP), when emergencies arise in passenger railways [17]. In this study, an integrated on-line optimization model based on a discrete-event simulation model is proposed with the objective of assigning vehicles from other transport lines to the disrupted lines, aiming at reducing the users' total time in system. The applicability of the two greedy heuristics solution approach was tested on a real data of Spanish National Railway company. Using predictive simulations, the first first heuristic (GH1) provides a good estimates of reassignment decision. Due to a high computational cost, GH1 was modified into the second greedy heuristics (GH2) with less number of predictive simulations. Although GH2 yields a better real-time reassignment decisions with faster response as compared to GH1, the accuracy of the decision made is questionable as there is a possibility that GH2 is being oversimplified.

Another work on real time decision and control in timetable disturbance was done by D'Ariano [18] who developed an advanced real-time train dispatching support system, called Railway traffic Optimization by Means of Alternative graphs (ROMA). As the research objective is to minimize the train consecutive delays at selected points in the case of service disruption, the efficient use of the infrastructure capacity at a network scale is able to increase punctuality during operations. The scheduling algorithms involved are rule-based dispatching and innovative global conflict resolution methods based on greedy heuristics, a lower bound and exact method. A B&B algorithm with fixed train routes provides a new feasible solution with a short computing time, besides compatible to the real-time traffic situation. This work has been a significant breakthrough in the rail disruption management as the ROMA is capable of being an efficient tool for real-time traffic management, even in the case when the actual timetable is in conflict or deadlock.

Goverde [19] presented a delay propagation model to compute the propagation of initial delays over a periodic railway timetable. The types and behaviors of delay propagations are explicitly analyzed. Structural and initial delays are also treated differently by means of a decomposition method. The algorithm which is implemented as a decision support system, namely Performance Evaluation of Timed Events in Railways (PETER) software, is able to be utilized in real-time train dispatching due to its fast processing time when tested on Dutch national railway timetable.

3 Solution Approaches

We seek to analyze these quantitative models more holistically by looking at the interdependence of the entire models with their solution approaches, in order to facilitate the understanding of rail service delay management. Table II summarizes the quantitative models of past studies related to railway rescheduling and the respective solution methods involved.

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In contrast to the exact methods, it is noted that the local search method which usually gives near optimal solutions are preferred in solving the models formulated. Among them, heuristics and B&B are the most popular solution approaches used in both mathematical and non-mathematical programming models of railway service delays problems.

TABLE II

SUMMARY OF QUANTITATIVE MODELS USED IN MINIMIZING RAILWAY SERVICE DELAYS

				Solution Approaches							
				Local Search Methods					**Exact Methods**		
	Author(s)	**Year**	**Models**	**Tabu Search**	**Heuristics**	**Branch & Bound**	**Branch & Cut**	**Meta-heuristics**	**Decomposition**	**Column generation**	**SAPI**
			Mathematical Programming								
1	Fekete et al.	2011	IP				√				
2	Kliewer & Suhl	2011	DIS		√						
3	Sato and Fukumura	2010	IP							√	
4	Acuna Agost	2010	MIP								√
5	Acuna Agost et. al.	2009	MIP			√	√				
6	Anita Schobel	2009	IP		√	√					
7	Rezanova and Ryan	2009	SP			√					
8	Sato et al.	2009	IP		√						
9	Holger Flier et. al	2008	IP		√	√					
10	Schachtebeck and Schobel	2008	IP		√						
11	Tornquist and Persson	2007	MIP			√					
12	Tornquist and Persson	2005	MIP	√				√			
13	Anita Schobel	2001	MIP			√					
			Non-Mathematical Programming								
1	Almodovar and Garcia	2011	SIM		√						
2	Rob M.P. Goverde	2010	DP						√		
3	Andrea D'Ariano	2010	RD			√					
4	Garcia et al.	2009	SIM		√						
5	Corman et. al.	2009	JSS			√					
6	Norio et. al.	2005	PERT					√			

Note: IP-Integer Programming, DIS-Dispatching Strategies, SP-Set Partitioning Problem Model, MIP-Mixed Integer Programming, SIM-Simulation Model, DP-Delay Propagation Model, RD-Rule-based Dispatching Model, LR-Linear Regression, JSS-Job Shop Scheduling Model, PERT-Project Evaluation & Review Technique.

Railway rescheduling involves real-time alteration of train schedules in a railway network which is highly interconnected. Mathematically, this is considered as a difficult, combinatorial and strongly constrained problem. The model's constraints require a large number of hard (operational) constraints and soft (desirability) constraints and the complexity of problem increases with the number of decision variables and constraints. Modeling and solving this railway rescheduling problem is thus considered a highly complex task and an NP-hard problem. In solving such

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complicated models with huge numbers of variables and constraints, aiming to get optimal solutions is something that could be very difficult and time-consuming. The method proposed might not be practical enough if it takes a long computing time, since railway rescheduling problems are supposed to be solved within seconds, in some cases. This is why many studies encountered have chosen approximation methods rather than exact solution approaches in generating their results.

In spite of the list of solution approaches presented in this review paper, there are many other mathematical solution approaches that are rarely used in solving train rescheduling problems. Some of them are a single method by itself, whereas the others are the integration of one or more techniques.

An uncommon solution approach called the Interior Point – Branch and Bound (IP-B&B) method were proposed by Elhedhli and Goffin [20]. The method integrates interior-point methods, decomposition techniques and B&B approaches to solve large scale MIPs.

The interior point method (IPM), which is a more powerful alternative to the Simplex method, was proposed by Karmarkar [21]. Freund and Mizuno [22] stated that IPM has permanently changed the landscape of mathematical programming theory, practice and computation. For over two decades, the IPM has been widely implemented to solve large scale linear programming and combinatorial optimization problems.

Branch & Price (B&P) integrates B&B and Column Generation method. B&P is a generalization of the B&B by strengthening linear programming relaxations with new inequalities throughout the branch-and-bound tree and focusing on column generation instead of row generation.

The rarely-used decomposition method which has only been utilized by Goverde [19] is actually an iterative process that divides the problems into sub-problems and solves them at each step of the iterations. The iteration processes would continue until ultimately the overall problem is solved. The method takes advantage of the nature of some large scale problems with structure of an interrelated system such as block angular or network structure to develop efficient computational procedures.

4 Conclusion

The criticisms on railway companies for the inefficiencies in rescheduling strategies and the impact have been the motivation behind this review paper. This paper provides the overview of the quantitative railway rescheduling techniques used in past research of delay management, focusing on mathematical and non-mathematical models. As train punctuality is a very important attribute of the public transport system, therefore it is a major challenge for railway operators to cope with the problem and look for new solutions and approaches which are both practical and economical in improving the service punctuality. To assist them, this review paper provide**s** a gap analysis for researchers to come up with a better rescheduling model and minimize the railway service delays.

This paper is a part of the study that will be conducted on the train rescheduling model to cope with the railway service disruptions within the Malaysia commuter rail system. In this context, the review justifies the need to develop an enhanced or hybrid mathematical model that could minimize service delays for the passenger trains. Thus, solution approaches to the mathematical model are to be proposed to the railway operator. These model**s** and the solution**s** approach will be presented in next papers of our study in the near future.

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